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III.

GRAPHICAL INTEGRATION.

By Edward C. Pickering.

Presented, Oct. 13, 1874.

When determining the relation between two physical quantities, we sometimes are able to measure only the relative rates at which they alter, instead of the alterations themselves. Or, to speak mathematically, if $y = f(x)$, instead of measuring various corresponding values of x and y, we can obtain only the values of x and $\frac{dy}{dx} = f'(x)$. Of course, if the form of $f'(x)$ is known, the ordinary methods of integration give $f(x)$ and y. But in general this is not given, and the usual methods of approximation are liable to introduce large errors, since by the summation the error adds, and the deviation continually becomes greater and greater. The problem is perhaps better understood by some familiar examples. Thus, given the velocity of the wind at certain times, to determine its total distance travelled per hour; given the velocity of a river, at various points of its cross-section, to find its total discharge; given the strength of an electric current, to find the total quantity transmitted. The case which actually suggested this problem was in calibrating a thermometer tube, having given the length of a mercury column at various points in the tube, to determine the correction to be applied for unequal diameters of the tube at various points. Here the various lengths of the column give the values of $\frac{dy}{dx}$, and the distance of its centre from one end gives the corresponding values of x. Construct a curve with these two quantities as coordinates, and the area included between this curve and the axis of X serves to measure the true values of y. To determine this area, draw a number of equidistant ordinates, and read from the curve the length of each. Then compute by Simpson's formula, $A = \frac{1}{3} a [(y_0 + 4 y_1 + y_2) + (y_2 + 4 y_3 + y_4) + \text{\&c.}]$, the area included between each second ordinate, the curve, and the axes. It gives the ordinate of points of the required curve, $y = f(x)$, the abscissa being of course that of the limiting ordinate.

To test these principles by an actual example, the following method was employed. A smooth curve was drawn by a pencil on a sheet of paper divided into squares, and the co-ordinates of six points on it noted as follows: $x = 0.7$, $z = .84$; $x = 2.3$, $z = 1.14$; $x = 4.4$, $z = 1.65$; $x = 5.8$, $z = 2.05$; $x = 7.6$, $z = 2.69$; $x = 9.6$, $z = 3.54$. It was then assumed that by some measurement these observations had been obtained, and that while x represented one of the variables z gave the relative rate of change, or $\frac{dy}{dx}$. These points were then laid off on a fresh piece of paper, and a smooth curve drawn through them. Of course this should agree with the original curve, were there no errors; and the deviation serves to show the amount of error to be expected. To obtain two independent results, a third curve was constructed, like the second, on another piece of paper. The values of z for $x = 1, 2, 3$, &c., were then determined on curves two and three, with results given in Table I., columns two and three. Applying Simpson's formula gives the numbers in columns four and five, which it will be seen agree very nearly, the difference being but little more than the errors of observation. Of course, if necessary, still closer results could be obtained, by residual curves and other methods; but in general the accidental errors present in the original observations render this refinement unnecessary.

TABLE I.

x	z'	z''	y'	y''	Δ
0	.73	.72	0.000	0.000	.000
1	.87	.87			
2	1.08	1.06	.882	.877	—.005
3	1.30	1.30			
4	1.54	1.54	2.185	2.177	—.008
5	1.82	1.82			
6	2.12	2 11	4.075	4.065	—.010
7	2.46	2.47			
8	2.84	2.82	6.542	6.533	—.009
9	3.26	3.26			
10	3.77	3.77	9.816	9.805	—.009

As another example, suppose the following measurements made in calibrating a thermometer tube: $x = 5°$, $z = 10°.0$; $x = 28°$, $z = 10°.4$; $x = 54°$, $z = 10°.7$; $x = 83°$, $z = 10°.9$, in which z gives the length of the mercury column, and x the position of its middle point.

The problem is to determine the correction to be applied to the observed temperatures, assuming the 0° and 100° points to be correct. Constructing a curve with the co-ordinates given above, we deduce the points given in columns one or two of Table II. Now, calling m the volume of the mercury drop, we have $z : m = dx : dv$, or $\frac{dv}{dx} = \frac{m}{z}$. Hence, we must use for ordinates in our summation the reciprocal of z as given in column three. Treating these as before, we obtain by the formula column four, and dividing by the total sum 283.8 gives in column five the true temperature, and subtracting the observed readings from these gives the correction in column six.

TABLE II.

t°	L	$\frac{dv}{dx}$	v	t'°	Δ
0	9.9	10.10	0.0	0°.00	0°.00
10	10.1	9.90			
20	10.3	9.71	59.4	20°.93	0°.93
30	10.4	9.62			
40	10.5	9.52	117.1	41°.26	1°.26
50	10.6	9.43			
60	10.7	9.35	173.3	61°.06	1°.06
70	10.8	9.26			
80	10.9	9.17	228.9	80°.65	0°.65
90	10.9	9 17			
100	11.0	9.09	283.8	100°.00	0°.00

To determine how rapidly the errors diminish, increasing the number of ordinates, the area included between the axis and the curve $y = \frac{1}{\pi} \sin x$ was computed for 2, 4, 6, 12, and 18 divisions; the errors in these cases were .030047, .001454, .000276, .000019, .000003, so that a high degree of accuracy is readily obtained. M. Chevilliet has recently shown (*Comptes Rendus*, lxxviii. p. 1841) that the error in Simpson's formula depends on $\frac{h^4}{180} \frac{d^3 y}{d x^3}$, while the method of summing by trapeziums gave $\frac{h^2}{12} \frac{dy}{dx}$. In an example he finds that the area of the curve $x \log x$, between $x = 10$ and $x = 20$, is given correctly by Simpson's formula, taking ten intervals, within .000005, while by the method of trapezoids the error is .001809. Evidently, then, it is easy to obtain by the first of these formulas as great an accuracy in the result as is needed in almost any physical research.